# Explicit Control of Topological Evolution in 3D Mesh Morphing

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Existing 3D mesh-morphing techniques are often limited to cases in which 3D source meshes to be morphed are topologically equivalent. Previously, we published a 3D mesh-morphing scheme based on interpolation of 3D source meshes by using a 4D tetrahedral mesh.[1] While the algorithm could potentially morph source meshes of different topological types, it lacked an effective way to explicitly control evolution of topology. This sketch presents a new approach to explicitly specifying a path of topological evolution while morphing 3D meshes of different topological types. The formalism we employed, while concise, is expressive enough to precisely specify all the possible topological changes that could occur during such topology-altering mesh morphing.

## The Formalism

Our shape-morphing algorithm directly interpolates 3D meshes by using a 4D tetrahedral mesh (a discretized version of 4D hyper-surface), which is embedded in 4D space spanned by x, y, z and t(time)-axis.[1] If a topological evolution occurs during a morph, it occurs at a critical point of the 4D hyper-surface that interpolates the source 3D surfaces. According to Fomenko, et al.,[2] any such topological evolution can be invoked by attaching one of four types of topological handles, listed in Figure 1, to the 3D surfaces involved. In the figure, the shaded part of each handle is "glued" to an existing surface and then eliminated. Careful examination of 4D hyper-surface (and their embedding in 3D space) shows that all the possible transitions in topological evolution are listed in Figure 2.


Figure 1. Four topological handles.

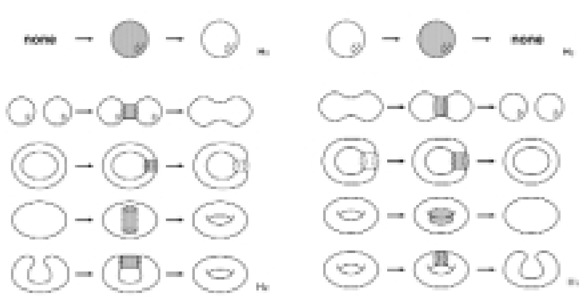


Figure 2. List of possible topological transitions.

## The Mechanism

To explicitly specify a topological transition, we insert a topological "keyframe," a concrete version of one of the topological transitions above. A keyframe is a special mesh that consists of two meshes that are geometrically identical but topologically different. Two topologically different meshes in a keyframe relate 3D source meshes of different topological types. Obviously, it is not possible to prepare an infinite number of keyframes for every combination of complex source meshes and topological transitions. Instead, we prepare keyframes in their simplest forms.

As stated above, our algorithm morphs 3D source meshes by interpolating them with a 4D tetrahedral mesh.[1] To simplify the task of creating the interpolator 4D tetrahedral mesh, our algorithm starts from a set of simplified source meshes to create a coarse tetrahedral mesh. The tetrahedral mesh and source meshes are then refined so that the original details are regained in the source meshes. Taking advantage of this framework, we prepare and insert the key frames only at the coarsest of the resolution levels.

As an example, Figure 3 shows creation of two "faces" of a keyframe that relates a simplified torus and a simplified sphere. In the top figure, the hole of a torus is shrunk to a single critical point to make one "face" of a keyframe topologically equivalent to a torus. In the bottom figure, another "face" of the key frame, a mesh topologically equivalent to a sphere, is created by plugging the hole with a topological handle H_1.

Interpolating the keyframe with a pair of (simplified) source meshes of different topological types creates a 4D tetrahedral mesh as shown in Figure 4. Mesh refinement and 3D shape extraction create smooth shape-morphing sequences as shown in Figure 5.



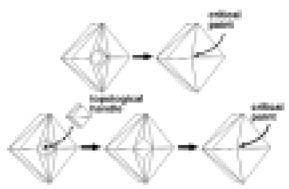


Figure 3. Keyframe generation.

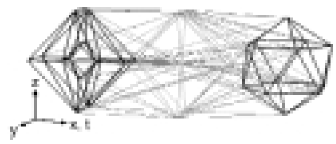


Figure 4. Tetrahedral mesh. (The t and x axes are overlaid.)

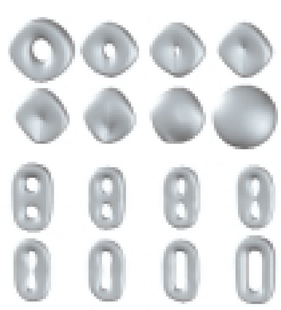

Figure 5. Results.

*References*
1. Ohbuchi, R., Kokojima, Y., & Takahashi, S. (2001). Blending shapes by using subdivision surfaces. *Computers and Graphics, 25* (1), 41-58.
2. Fomenko, A.T. & Kunii, T. L. (1997). *Topological modeling for visualization*. Springer-Verlag, 105-125.